



## **General Certificate of Education**

# **Mathematics 6360**

**MPC4      Pure Core 4**

## **Mark Scheme**

*2007 examination - June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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## Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
✓ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## MPC4

Q	Solution	Marks	Total	Comments
1(a)	$2\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 3 = -3$	M1A1	2	use of $\pm\frac{1}{2}$ SC NMS -3 1/2 No ISW, so subsequent answer "3" AO
	<p><b>Alt</b> algebraic division:</p> $\begin{array}{r} x \\ 2x+1 \overline{) 2x^2 + x - 3} \\ \underline{2x^2 + x} \phantom{-3} \\ -3 \end{array}$ <p><b>Alt</b> <math>\frac{x(2x+1)-3}{2x+1}</math></p>	(M1) (A1)	(2)	complete division with integer remainder remainder = -3 stated, or -3 highlighted
(b)	$\frac{(2x+3)(x-1)}{(x+1)(x-1)}$	B1 B1	3	numerator } not necessarily in fraction denominator }
	$= \frac{2x+3}{x+1}$	B1		
(b)	<b>Alternative</b> $\frac{2x^2 - 2 + x - 1}{x^2 - 1}$			
	$= 2 + \frac{x-1}{x^2-1}$	(M1)		
	$= 2 + \frac{x-1}{(x-1)(x+1)}$	(B1)		
	$= 2 + \frac{1}{x+1}$	(A1)	(3)	CAO in this form. Not $\frac{2x+3}{x+1} \frac{x-1}{x-1}$
<b>Total</b>			<b>5</b>	

## MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$(1+x)^{-1} = 1 + (-1)x + px^2 + qx^3$	M1	2	$p \neq 0, q \neq 0$
	$= 1 - x + x^2 - x^3$	A1		SC 1/2 for $= 1 - x + px^2$
(ii)	$(1+3x)^{-1} = 1 - 3x + (3x)^2 - (3x)^3$	M1	2	$x$ replaced by $3x$ in candidate's (a)(i); condone missing brackets
	$= 1 - 3x + 9x^2 - 27x^3$	A1		CAO SC $x^3$ -term : $1 - 3x + \frac{3}{9}x^2$ 1/2
(b)	<b>Alt (starting again)</b> $(1+3x)^{-1} = 1 - (3x) +$ $\frac{(-1)(-2)(3x)^2}{2!} + \frac{(-1)(-2)(-3)(3x)^3}{3!}$	(M1)	(2)	condone missing brackets accept 2 for 2!, 3.2 for 3!
	$= 1 - 3x + 9x^2 - 27x^3$	(A1)		CAO
	$\frac{1+4x}{(1+x)(1+3x)} = \frac{A}{1+x} + \frac{B}{1+3x}$	M1		correct partial fractions form, and multiplication by denominator
	$1+4x = A(1+3x) + B(1+x)$			
	$x = -1, x = -\frac{1}{3}$	m1		Use (any) two values of $x$ to find $A$ and $B$
	$A = \frac{3}{2}, B = -\frac{1}{2}$	A1		$A$ and $B$ both correct
	<b>Alt:</b> $\frac{1+4x}{(1+x)(1+3x)} = \frac{A}{1+x} + \frac{B}{1+3x}$	(M1)		correct partial fractions form, and multiplication by denominator
	$1+4x = A(1+3x) + B(1+x)$			
	$A+B=1, 3A+B=4$	(m1)		Set up and solve
	$A = \frac{3}{2}, B = -\frac{1}{2}$	(A1)		$A$ and $B$ both correct
(c)(i)	$\frac{1+4x}{(1+x)(1+3x)} = \frac{3}{2(1+x)} - \frac{1}{2(1+3x)}$	M1	3	multiply candidate's expansions by $A$ and $B$ , and expand and simplify
	$= \frac{3}{2}(1-x+x^2-x^3) - \frac{1}{2}(1-3x+9x^2-27x^3)$	m1		CAO
	$= 1 - 3x^2 + 12x^3$	A1		SC $A$ and $B$ interchanged, treat as miscopy. $(1 - 4x + 13x^2 - 40x^3)$
	<b>Alt:</b> $= \frac{1+4x}{(1+x)(1+3x)} = (1+4x)(1+x)^{-1}(1+3x)^{-1}$	(M1)		write as product, using expansions condone missing brackets on $(1+4x)$ only
(ii)	$= (1+4x)(1-x+x^2-x^3)(1-3x+9x^2-27x^3)$	(m1)	attempt to multiply the three expansions up to terms in $x^3$	
	$= 1 - 4x + 13x^2 - 40x^3 + 4x - 16x^2 + 52x^3$	(m1)	CAO	
	$= 1 - 3x^2 + 12x^3$	(A1)	OE and nothing else incorrect	
	$ x  < 1$ and $ 3x  < 1$	M1	OE Condone $\leq$	
	$ x  < \frac{1}{3}$ (0.33)	A1	2	
<b>Total</b>			<b>12</b>	

## MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$R = 5$ $\tan \alpha = \frac{3}{4}$ (OE) $\alpha = 36.9^\circ$ (ISW 216.9)	B1 M1A1	3	SC1 $\tan \alpha = \frac{4}{3}$ , $\alpha = 53.1^\circ$ $R, \alpha$ PI in (b)
(b)	$\cos(x - \alpha) = \frac{2}{R}$ $x - \alpha = 66.4^\circ$ $x = 103.3^\circ$ $x = 330.4^\circ$	M1 A1 A1F A1F	4	accept $330.5^\circ$ , -1 each extra ft on acute $\alpha$
(c)	minimum value = -5 $\cos(x - 36.9) = -1$ $x = 216.9^\circ$	B1F M1 A1	3	ft on $R$ SC $\cos(x + 36.9)$ treat as miscopy 216.9 or better accept graphics calculator solution to this accuracy SC Find max: max = 5 at $(x + 36.9)$ stated 1/3  Max 8/10 for work in radians
	<b>Total</b>		<b>10</b>	

## MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$t = 0: x = 3$	B1	1	
(ii)	$t = 14: x = 15 - 12e^{-1}$ $= 10.6$	M1 A1	2	or $15 - 12e^{-\frac{14}{14}}$ CAO
(b)(i)	$-5 = -12e^{-\frac{t}{14}}$	M1		substitute $x = 10$ ; rearrange to form $p = qe^{-\frac{t}{14}}$
	$\ln\left(\frac{5}{12}\right) = -\frac{t}{14}$ (OE)	m1		take lns correctly
	$t = 14\ln\left(\frac{12}{5}\right)$	A1	3	must come from correct working
(ii)	$t = 12.256... \approx 12$ days	B1F	1	ft on $a, b$ if $a > b$ ; accept $t = 12$ NMS Accept 12 from incorrect working in b(i) Accept 13 if 12.2 or 12.3 seen
(c)(i)	$\frac{dx}{dt} = -\frac{1}{14} \times -12e^{-\frac{t}{14}}$	M1		differentiate; allow sign error condone $\frac{dy}{dx}$ used consistently
	$= -\frac{1}{14}(x-15)$	m1		Or $\frac{1}{14}\left(12e^{-\frac{t}{14}}\right)$ and $12e^{-\frac{t}{14}} = 15 - x$ seen
	$= \frac{1}{14}(15-x)$	A1	3	AG – be convinced CSO
	<b>Alt:</b> $t = -14\ln\left(\frac{15-x}{12}\right)$	(M1)		attempt to solve given equation for $t$
	$\frac{dt}{dx} = \frac{-14\left(-\frac{1}{12}\right)}{\left(\frac{15-x}{12}\right)}$	(m1)		differentiate wrt $x$ , with $\frac{1}{15-x}$ seen; OE $\frac{1}{12}$
	$\frac{dt}{dx} = \frac{14}{15-x} \Rightarrow \frac{dx}{dt} = \frac{1}{14}(15-x)$	(A1)	(3)	AG – be convinced
	<b>Alt: (backwards)</b> $\int \frac{dx}{15-x} = \int \frac{dt}{14} = \pm 14\ln(15-x) = t + c$	(M1)		
	Use (0,3): $-14\ln(15-x) + 14\ln 12 = t$	(m1)		
	Solve for $x$ : $x = 15 - 12e^{-\frac{t}{14}}$	(A1)	(3)	All steps shown
(ii)	rate of growth = 0.5 (cm per day)	B1	1	Accept $\frac{7}{14}$
	<b>Total</b>		<b>11</b>	

## MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$x = 1, 5a^2 - a - 4 = 0$ $(5a+4)(a-1) = 0, a = 1$	M1 A1	2	condone $y$ for $a$ AG – be convinced, both factors seen or $a = -\frac{4}{5}$ or $1 \Rightarrow a = 1$ A0 for 2 positive roots (substitute $(1, 1) \Rightarrow 5 = 5$ no marks)
(b)	$\frac{dy}{dx} + 4$  $= 10xy^2 + 10x^2y \frac{dy}{dx}$  $x = 1, y = 1 \quad \frac{dy}{dx} + 4 = 10 + 10 \frac{dy}{dx}$  $\frac{dy}{dx} = -\frac{6}{9} = \left(-\frac{2}{3}\right)$ <b>Alt (for last two marks)</b> $\frac{dy}{dx} = \frac{10xy^2 - 4}{1 - 10x^2y}$  $(1,1) \Rightarrow \frac{10-4}{1-10} = -\frac{6}{9}$	B1B1  M1 M1 A1 M1 A1	7	(Ignore ' $\frac{dy}{dx}$ ' if not used, otherwise loses final A1) attempt product rule, see two terms added chain rule, $\frac{dy}{dx}$ attached to one term only condone $5 \times 2$ for 10 two terms, or more, in $\frac{dy}{dx}$  CSO  find $\frac{dy}{dx}$ in terms of $x, y$ and substitute $x = 1, y = 1$ must be from expression with two terms or more in $\frac{dy}{dx}$
(c)	$\frac{y-1}{x-1} = -\frac{2}{3}$ (OE)	B1F	1	ft on gradient ISW after any correct form
<b>Total</b>			<b>10</b>	



## MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\frac{dx}{d\theta} = -\sin \theta$ $\frac{dy}{d\theta} = 2 \cos 2\theta$	B1 B1	2	
(ii)	$\frac{dy}{dx} = -\frac{2 \cos 2\theta}{\sin \theta}$ , $\frac{dy}{dx} = -\frac{2 \cos \frac{\pi}{3}}{\sin \frac{\pi}{6}} = -2$	M1		use chain rule <del>their <math>\frac{dy}{d\theta}</math></del> and <del>their <math>\frac{dx}{d\theta}</math></del>
(b)	$y = 2 \sin \theta \cos \theta = 2\sqrt{1 - \cos^2 \theta} \cos \theta$	A1	2	substitute $\theta = \frac{\pi}{6}$
	$y = 2\sqrt{1 - x^2} x$	B1		use $\sin 2\theta = 2 \sin \theta \cos \theta$
	$y^2 = 4x^2(1 - x^2)$	B1		use $\sin^2 \theta = 1 - \cos^2 \theta$
	<b>Alt</b>	M1		$\sin \theta, \cos \theta$ in terms of $x$
	$y^2 = \sin^2 2\theta = (2 \sin \theta \cos \theta)^2$	A1	4	all correct CSO
$= (4) \sin^2 \theta \cos^2 \theta = (4)(1 - \cos^2 \theta) \cos^2 \theta$	(B1)		use of double angle formula	
$= (4)(1 - x^2)x^2$	(B1)		use of $s^2 + c^2 = 1$ to eliminate $\sin \theta$	
$= 4(1 - x^2)x^2$	(M1)		Substitute $\cos \theta$ for $x$	
		(A1)	(4)	CSO
	<b>Total</b>		<b>8</b>	

## MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = 3 - 6 + 3 = 0$ $= 0 \Rightarrow \text{perpendicular}$	M1 A1	2	attempt at sp, 3 terms, added $= 0 \Rightarrow$ perpendicular seen (or $\cos \theta = 0 \Rightarrow \theta = 90^\circ$ ) Allow $\frac{3}{-6}$ but not $\begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix} = 0$ $\frac{3}{0}$
(b)	$8 + 3\lambda = -4 + \mu$ $6 - 3\lambda = 2\mu$ $-9 - \lambda = 11 - 3\mu$ $\lambda = -2, \mu = 6$ verify third equation intersect at $(2, 12, -7)$ <b>Alt (for last two marks)</b> substitute $\lambda$ into $l_1$ and $\mu$ into $l_2$ intersect at $(2, 12, -7)$ , condone $\begin{pmatrix} 2 \\ 12 \\ -7 \end{pmatrix}$	M1 m1 A1 m1 A1 (m1) (A1)	5	set up any two equations solve for $\lambda$ and $\mu$ substitute $\lambda, \mu$ in third equation CAO $(2, 12, -7)$ found from both lines Note: working for (b) done in (a): award marks in (b)
7(c)	$\overline{AP} = \begin{pmatrix} 6 \\ 12 \\ -18 \end{pmatrix}$ $AP^2 = 504$ $AB^2 = 2AP^2$ $AB = 12\sqrt{7}$	M1 A1F M1 A1	4	$\overline{AP} = \pm \left\{ \text{their } \overline{OP} - \begin{pmatrix} -4 \\ 0 \\ 11 \end{pmatrix} \right\}$ ft on $P$ Calculate $AB^2$ OE accept 31.7 or better
	<b>Total</b>		<b>11</b>	

## MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\int \frac{1}{\sqrt{1+2y}} dy = \int \frac{1}{x^2} dx$	M1		attempt to separate and integrate
	$\int \frac{1}{\sqrt{1+2y}} dy = k\sqrt{1+2y}$	m1		
	$\sqrt{1+2y} = -\frac{1}{x} (+c)$	A1		OE A1 for $\sqrt{1+2y}$ depends on both Ms
	$x=1, y=4 \Rightarrow c=4$	A1		A1 for $-\frac{1}{x}$ depends on first M1 only
		m1		+c must be seen on previous line
		A1F	6	ft on $k$ and $\pm\frac{1}{x}$ only
(b)	$1+2y = \left(4 - \frac{1}{x}\right)^2$	m1		need $k\sqrt{1+2y} = 'x$ expression with + c'
	$2y = 15 + \frac{1}{x^2} - \frac{8}{x}$	A1	2	<b>and</b> attempt to square both sides terms on RHS in any order AG – be convinced CSO
	<b>Total</b>		<b>8</b>	
	<b>TOTAL</b>		<b>75</b>	